Cross-impact balances:
A system-theoretical approach to cross-impact analysis

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Abstract

Cross-Impact methods are standard tools of the scenario technique. They provide a number of structured processes for the deduction of plausible developments of the future in the form of rough scenarios and are based on expert judgments about systemic interactions. Cross-Impact methods are mostly used for analytical tasks which do not allow the use of theory-based computational models due to their disciplinary heterogeneity and the relevance of “soft” system knowledge, but on the other hand are too complex for a purely argumentative systems analysis. The essentials of a new Cross-Impact approach (Cross-Impact Balance Analysis, CIB) are outlined; it is of high methodological flexibility and is especially suitable for the use in expert discourses due to its transparent analytical logic. Due to its mathematical qualities it is also particularly well suited for the analytical integration of calculable system parts. An application of CIB to a project on the generation of electricity and climate protection is described. This explicates that CIB scenarios correspond to the solutions of slowly time-varying pair-force systems.

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Keywords: Cross-Impact; System; Theory; Scenario; Analysis

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1. Introduction

Scenario analyses are an often-used tool in long-term planning. In contrast to prognoses they provide a suitable handling of the uncertainty context of strategic decisions in complex situations. Surveys demonstrate that most large enterprises meanwhile have turned to using scenario analyses after a time of initial hesitation [1–5]. The overwhelming part of the enterprises which had already had some experience with scenario analyses evaluated the method as being helpful or even very helpful respectively essential. Scenarios nowadays play an indispensable role in political consulting, as well. Scenarios are [6]

“an internally consistent view of what the future might turn out to be—not a forecast, but one possible future outcome”.

The key to their importance is contained in the notion of “internal consistency”. Long-term planning often demands an understanding of multidisciplinary connections, for example the interdependence of demographic, social, technological, economic and political developments. For each separate area the relevant specialists may supply prognoses, e.g. in a form as follows:

1. The gross domestic product of the year \( n \) will be \( x_1 \) for the lower variant, \( x_2 \) for the middle variant and \( x_3 \) for the upper variant.
2. The proportion of the consumer-oriented materialistic part of the population in the year \( n \) will be \( y_1 \) for the lower variant, \( y_2 \) for the middle variant and \( y_3 \) for the upper variant.
3. The market share of the consumer technology \( A \) being developed today will be \( z_1 \) for the lower variant, \( z_2 \) for the middle variant and \( z_3 \) for the upper variant in the year \( n \).
4. 

However, more is needed for strategic decisions. They demand an overall image that expresses in which combinations the lower, middle or upper variants of the disciplinary prognoses can possibly occur, because it is clear that for such quantities as have been mentioned above the variants do not occur or fail to occur independently of each other. So strategic decisions often need a multidisciplinary analysis of the correlations between the relevant quantities. The results are scenarios, and their claim to internal consistency demands considerable insight into the nature of the connections between the relevant problem areas.

That is of course where the difficulty lies. There are some well-structured planning tasks for which a convincing mathematical modelling is possible. In these cases all interactions can be captured by model computations. But these planning tasks unfortunately are rather exceptions than representing the rule: in many cases a well-founded mathematical modelling is only possible if one is willing to except important parts of the problem and to limit an originally comprehensive understanding of the problem to the computable part of the problem. The price that has to be paid for this is the risk to fail [7].

The methodical antithesis to theory-based mathematical modelling is verbal analysis (e.g. “Intuitive Logics”, compare [8]). It is often used in scenario analyses and can produce impressive results. But since the human mind is limited in its capability of mentally processing multifactor-interdependencies, a verbal analysis is not very suitable for the analysis of highly complex problems.

On the other hand, the capturing of interactions is a particular strength of “System Dynamics” [9,10]. Interactions are modelled by difference equations and integrated to capture their temporal development.
The use of “System Dynamics” requires, though, that the interactions between the elements of the system are known as detailed as necessary to depict them as formulas.

The question remains on how we can develop an understanding of systems:

- for which no quantitative theory as a basis for a well-founded mathematical modelling is available, and
- whose interactions are too complex to understand the system intuitively, and
- for which the available knowledge about the interrelations of the system is partly or wholly too qualitative to be expressed trustworthily by a mathematical formula.

There is no doubt that many practically relevant planning and decision tasks fall into this category. There can be no doubt as well that the analysis of such weakly structured problems can only produce rough scenarios, but no detailed results. This cannot be changed and is the natural result of the limited knowledge about the system. But as an aid to orientation even rough and rather qualitative insights into possible developments of the future can be of considerable help to the choice of suitable strategies.

Well-known for many years, the Cross-Impact analysis is a family of methods that has been developed into many variants to generate rough scenarios for complex, but weakly structured systems. Its approach is based on the evaluation of interrelations between the most important influential factors in a system by experts who evaluate pairs of these factors (for example as conditional probabilities), and then to find out which scenarios are probable in view of the established network of interrelations with the help of suitable mathematical procedures. The fact that this method is based on expert judgments makes it possible to use it also for weakly structured problems; on the other hand, the results depend fundamentally on the involved experts’ ability to evaluate the system and the relations between its elements. The status of the Cross-Impact analysis resulting from the described concept can hardly be summarized more appropriately and shortly than by Olaf Helmer [11]:

“Cross-Impact analysis represents a schema for collating and systemizing [...] expert judgments, so as to make it possible to construct a conceptual substitute, however imperfect, for a wished-for but nonexistent theory of how events affect one another in a multidisciplinary context.”

2. Cross-Impact analysis: a short overview

The first approaches to Cross-Impact analysis were developed in the 1960s in response to a shortcoming of Delphi surveys. In these, experts were asked about the future chances of different technologies, but the mutual influence existing between the technologies was not taken into account. Gordon and Hayward therefore introduced a concept in 1968 saying that the occurrence of an event (for example the realization of a technology) modifies the occurrence probability of other events [12]. The coefficients according to which event \( x \) raises or lowers the occurrence probability of event \( y \) were called Cross-Impacts by Gordon and Hayward and have to be determined by expert judgments. A statement whether the Cross-Impact effects raise or lower the “naively” estimated event probabilities is received by means of a repeated randomly controlled simulation of the course of events.
Within a short time, the basic idea of Cross-Impact Analysis received great interest and in the following decade many different variants were developed and their usefulness was discussed, also partly critically [13,14]. The variants differ especially from each other in the following features:

- Sometimes experts—as in the case of Gordon and Hayward—are asked for an assessment how an event modifies the occurrence probability of another event. This involves the use of the concept of “causal probabilities”. In other cases experts are asked for conditional probabilities [15] or joint probabilities [16].
- Since the original concept of the Cross-Impact analysis dealt with the interactions between events, some authors discussed the interactions between trends [17,18] and between trends and events [11,19–21].
- Part of the developed Cross-Impact approaches, including the one by Gordon and Hayward, aims at the examination of binary variables, for example events that have occurred by a certain time or not. Other approaches use quantitative variables, which are discretized by intervals (for example [22–24]).
- The overwhelming part of the Cross-Impact approaches follows a probabilistic approach, which often finds expression in the choice of probability quantities as Cross-Impacts and in the determination of event probabilities and scenario probabilities as results. But there are deterministic forms of Cross-Impact Analysis as well, which lead to a conceptual propinquity to System Dynamics approaches [25–28]. Some structural analyses, for example MICMAC [29], can be counted among the group of deterministic Cross-Impact Analyses in a wider sense.

A large part of the methods research about Cross-Impact Analysis was done in the 1970s and the early 1980s. The mainstream of scenario analysis has been concentrating on the development and the use of large computational models, especially concerning economic problems and energy issues. Nevertheless, the interest and the continuing need for a methodical treatment of problems that are mathematically difficult to capture is revealed by several Cross-Impact application reports in the more recent past as well [30–32]. Newer efforts in methods research aim at the application of fuzzy-concepts to Cross-Impact methods [33–35], the avoidance of inconsistencies between marginal and scenario probabilities [24] and bridging to decision theory [36].

3. Some problems of cross-impact methods

In the following, a new approach to Cross-Impact Analysis is developed which overcomes some of the difficulties of earlier approaches and opens up new possibilities: one of the difficulties of many previous probabilistic Cross-Impact methods has been that they (among other things) demand the estimation of at least one of the following quantities by the judging experts: conditional probabilities or joint probabilities of event pairs, or the marginal probability of events. To be able to do these estimations properly, the experts not only need to know which interrelations exist in a system but they also have to recognize which results this impact network will produce.

So they have to be capable of a “mental integration” [17]. But exactly this is a task, as has been mentioned before, for which the human mind is ill equipped [37]. Basically, this means that at the beginning of an analysis the experts are expected to possess insights which rather should be the results of an analysis. It should be the aim of developing a method, however, to strive for a more promising division
of labour between man and method. Everybody should contribute that at which they are best, namely the expert at recognizing the impact pattern within a complex system and the mathematical method at analyzing how this impact pattern works. The method described in the following pursues this division of tasks and thereby increases the chance to get appropriate expert judgments about complex systems.

Another problem of many Cross-Impact methods are their calculation methods, which are partly mathematically demanding. They are not really difficult for mathematically trained analysts, but a ‘black box’ for many experts and often also for the consumers of the analysis. We have to bear in mind that the nature of the Cross-Impact analysis is really to analyze multidisciplinary connections, which typically means there are people involved from the non-mathematized sciences or practitioners without deep mathematical training. But the ability and the motivation to bring one’s knowledge into the analyses, to trust the results and to use them for decisions are not promoted by an ‘unfathomable’ analysis. Alter writes in his list of quality criteria for Cross-Impact methods [38]:

“In many generic cross-impact models the underlying mathematical structures are quite difficult to understand. Consequently, their computational routines are so difficult to explain that users are more or less expected to ‘have faith’ in the black box performing the cross-impact calculations. Another aspect of the problem is that complex computational forms make it very difficult to trace the impact of particular events or parameter values.”

The cross-impact method proposed in the following will stand out due to an especially good relation between its method transparency and its variety of statements. An important contribution to the transparency of the method will be that, though the result scenarios have to be designed by a computer, everybody will be able to verify them after that by a simple control calculation. As the core of the method is based on a balancing of the Cross-Impacts, it is called Cross-Impact Balance (CIB) in the following.

4. The Cross-Impact Balance Analysis

In order to describe the Cross-Impact Balance Analysis (CIB Analysis) process a simple example is used in the following. It is about the development of the price of oil and follows the one used by [22] but has been simplified and modified a bit. The evaluation method applied to the example below differs, however, basically from the one used by Honton et al. The example does not want to produce relevant statements with regard to the development of the oil price, but is only meant to provide an illustrative and manageable frame for the description of the method.

The preparatory step for the carrying out of the Cross-Impact Analysis is to collect and choose the most important factors which have a significant direct or indirect influence on the object of the examination. After that, it is established for these ‘descriptors’ which development variants could come into consideration for the target year of the analysis by means of literature search, model prognoses or expert judgments. For our example the result could look as shown in Table 1. The descriptors can have a quantitative nature and therefore their states can be numerical. But they can also have a qualitative nature and, according to that, may have linguistic characterizations for their states. The possibility to process both types of descriptors together is one of the advantages of all Cross-Impact methods. The choice of the states and their intervals shall be done in a way that all development possibilities judged as probable are included. The finer the intervals of the quantitative descriptors are chosen, the more detailed will be the results later on; on the other hand, the effort necessary to get the Cross-Impact judgments will rise greatly.
Our example uses 5 descriptors with overall 16 states. In practice, considerably larger systems are usually used for analyses. In three projects about the future of energy supply, in which experiences about the application of CIB have been collected, 9–15 descriptors and 24–43 states have been used.

The next step is to build up a matrix containing judgments which express the influence of each descriptor on each one of the other descriptors. These judgments are normally gained by asking experts. A crucial point in which Cross-Impact methods differ is the Cross-Impact-Question, a question, by which the experts are asked for their judgment on the individual interrelations. For some methods, e.g. the conditional probabilities are asked directly. For other methods, small integer numbers are asked which express how the probability of an event or a state changes as soon as another event or another state has occurred. In CIB, though, a slightly different quantity is gathered as Cross-Impact judgment. For the moment, this quantity shall be defined as the answer to the question

“If the only piece of information about the system is that Descriptor \( X \) has the state \( x \), will you evaluate this due to the direct influence of \( X \) on \( Y \) as a hint that Descriptor \( Y \) has the state \( y \) (promoting influence, positive points assessed) or as a hint that Descriptor \( Y \) has not the state \( y \) (restricting influence, negative points assessed)?”

The stronger one hint must be evaluated compared to other hints, the more points it is to be given. As the importance of the hints normally has to be estimated, it is usually agreed on doing the evaluation with small integer numbers, interpreting the row descriptors as source of influence, the column descriptors as target of influence. Only the direct influence shall be evaluated to avoid double counts (indirect influence, like e.g. \( X \) has an effect on \( Z \) and \( Z \) has an effect on \( Y \), is automatically taken into account by CIB). Does no direct influence exist, the Cross-Impact judgment “0” is given. As each hint in favor of one state is implicitly a hint against the alternative states, the sum of the Cross-Impact judgments is zero in each judgment group. The diagonal of the Cross-

<table>
<thead>
<tr>
<th>Descriptor</th>
<th>States</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. World GDP growth</td>
<td>&lt;2%/year</td>
</tr>
<tr>
<td></td>
<td>2–3%/year</td>
</tr>
<tr>
<td></td>
<td>&gt;3%/year</td>
</tr>
<tr>
<td>2. Borrowing industrial countries</td>
<td>low</td>
</tr>
<tr>
<td></td>
<td>medium</td>
</tr>
<tr>
<td></td>
<td>high</td>
</tr>
<tr>
<td>3. World tensions</td>
<td>strong</td>
</tr>
<tr>
<td></td>
<td>moderate</td>
</tr>
<tr>
<td></td>
<td>weak</td>
</tr>
<tr>
<td>4. Cohesion of OPEC</td>
<td>strong</td>
</tr>
<tr>
<td></td>
<td>moderate</td>
</tr>
<tr>
<td></td>
<td>weak</td>
</tr>
<tr>
<td>5. Oil price</td>
<td>&lt;20$</td>
</tr>
<tr>
<td></td>
<td>20–35$</td>
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<tr>
<td></td>
<td>35–50$</td>
</tr>
<tr>
<td></td>
<td>&gt;50$</td>
</tr>
</tbody>
</table>

Table 1
The example “oil price”: descriptors and their states
Impact matrix is left empty because the above stated Cross-Impact question is senseless for diagonal elements.

This definition of Cross-Impact judgments is quite heuristic. Usually it is sufficient in practical work for, as experience shows, it suffices to explain to the experts which kind of valuation is expected from them. For an evaluation of the methodological aspects of the process it is, however, desirable to develop a theoretical background for the thus emerging Cross-Impact judgments. This will be done in another paragraph of this paper later on.

Table 2 shows the Cross-Impact matrix of our example. Estimations by the author have been entered as Cross-Impact judgments. They are of an exemplary character and can be put into words as follows:

+3: strongly promoting direct influence
+2: promoting direct influence
+1: weakly promoting direct influence
0: no direct influence
−1: weakly restricting direct influence
−2: restricting direct influence
−3: strongly restricting direct influence.

If the grading of the strength ratios renders it necessary, bigger numbers can be used as well. If single interrelations are known in sufficient detail, also other than integer numbers can be used. The entry −3 in the row cohesion of OPEC strong and the column price of oil <20$ means that a strong cohesion of the OPEC was judged to be of a strongly restricting influence on the possibility of very low oil prices.

The principle of compensation can give some help to determine the judgments: two opposing influences on one state are to be judged as equally strong if their effects can compensate each other. If it is to be estimated that one of the influences predominates during a confrontation, this one shall be judged higher, i.e. be given a higher number. This simple rule establishes the foundation for the definition of the evaluation algorithm later on.

The CIB matrix can be understood as a hypermatrix, i.e. as a matrix whose elements are themselves matrices. These submatrices \( C_{ij} \) will be called judgment sections in the following text. A row of a submatrix is a judgment group, a single entry in this group is a judgment cell. Usually one will use integer numbers as entries, but the CIB algorithm does not require this limitation.

The benefit of the Cross-Impact matrix is that it helps to check all system states by enumeration if they are self-consistent (i.e. not contradictory) in the sense of the established understanding of the system. The check of a system state ("scenario") takes place in two steps which investigate the double role of the descriptors as source and target of influences. Scenarios that do not contain contradictions between both perspectives are then accepted as valid.

The role of the descriptors as a source of influences is examined by marking all those rows in the Cross-Impact matrix which are part of the scenario that is to be tested. Table 2 shows this for the arbitrarily chosen example of the scenario:

- world GDP growth >3%
- borrowing of the industrial nations low
- world tensions weak

\[ \text{In this case there are } 3 \times 3 \times 3 \times 3 \times 4 = 324 \text{ system states; in practice systems include more descriptors usually and several thousand or million system states are possible.} \]
Table 2
Cross-impact matrix of the “oil price” system

<table>
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<tr>
<th></th>
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<tbody>
<tr>
<td>&lt; 2 %/yr</td>
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<tr>
<td>2 – 3 %/yr</td>
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<tr>
<td>&gt; 3 %/yr</td>
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<table>
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<tr>
<th>2. Borrowing industrial countries</th>
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<tbody>
<tr>
<td>high</td>
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<tr>
<td>medium</td>
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<tr>
<td>low</td>
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<tr>
<th>3. World tensions</th>
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<tbody>
<tr>
<td>strong</td>
</tr>
<tr>
<td>moderate</td>
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<tr>
<td>weak</td>
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<tr>
<th>4. Cohesion OPEC</th>
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<tbody>
<tr>
<td>strong</td>
</tr>
<tr>
<td>moderate</td>
</tr>
<tr>
<td>weak</td>
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</table>

<table>
<thead>
<tr>
<th>5. Oil price</th>
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<tbody>
<tr>
<td>&lt; 20 $</td>
</tr>
<tr>
<td>20 – 35 $</td>
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<tr>
<td>35 – 50 $</td>
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<tr>
<td>&gt; 50 $</td>
</tr>
</tbody>
</table>

States according to test-scenario:

Impact balances:

Impact score of state “Borrowing medium”

Impact balance of descriptor “Cohesion OPEC”

Gray rows mark an arbitrary scenario example (“test-scenario”, cf. text).
- cohesion of OPEC strong
- oil price 20–35$

of which the states are emphasized by shade. In the rows that are marked all the weighted influences can be seen which follow from the occurrence of one state for the states of the other descriptors. Due to the fact that for each descriptor there are normally influences from several other descriptors, the influences have to be combined, i.e. balanced as defined by the principle of compensation. This is achieved by the impact balances (Table 2 beneath) in which the Cross-Impact judgements of the marked rows are summed up. The summation implements exactly the logic of the principle of compensation: contrary influences of the same strength compensate each other, contrary influences that vary in strength weaken each other by the prevalence of the stronger influence.

From this first step follows which effects the presented scenario has overall on each of the descriptors. For the second step we take the perspective of the descriptors as a target of these influences and consider which descriptor states there would have to be on account of the impact balances found. For the descriptor “world GDP growth” follows from the balance row in Table 2 that the influences in favor of the state “>=3%” predominate as it has the highest impact score in its impact balance. In the same way, the impact balances for the other descriptors described argue in favor of the realization of the states “borrowing of the industrial nations low”, “world tensions weak”, “cohesion of OPEC strong” and “oil price 35–50$”. With that we can evaluate this scenario which has been arbitrarily chosen as an example.

For this we compare the states that are recommended by the impact balances as plausible states because they show the highest impact score with the test scenario. For four out of five descriptors the input balances recommend exactly those states we have assumed in our test scenario. But for one descriptor there is a discrepancy between assumption and result: instead of the state “oil price 20–35$” the input balance of this descriptor refers to the state “oil price 35–50$”.

This is a classic paradox. From a hypothesis (the test scenario) conclusions can be drawn which directly contradict this hypothesis. So the examined test scenario in Table 2 is not suitable to evoke a self-consistent network of influences in the system. Therefore it is called an inconsistent scenario and rejected.

This kind of logic used for judging test scenarios in the CIB analysis is called principle of consistency. In the complete scan of all the 324 possible scenarios only those scenarios are described as valid (i.e. consistent) in which no discrepancy in the way shown arises for any descriptor. That means that in the consistent scenarios the arrow markings in the row “states according to test scenario” at the bottom of Table 2 perfectly correspond to the arrow markings in the row “states according to impact balance”.

Normally only few scenarios have this quality, to which therefore applies a particular inner consistency that distinguishes them from the mass of combinatory possibilities. Within the scope of the CIB analysis this small set of distinguished scenarios is used as an immediate result and as the object for further investigations. The list of all consistent scenarios for the example in Table 2 contains 3 scenarios; the other 321 scenarios show at least one inner discrepancy (compare Table 3). The number of consistent scenarios in this example is not untypical and demonstrates that the method works highly selective when it comes to a differentiation between ‘good’ and ‘bad’ scenarios.

3 The reason in terms of content for this is, as can be seen in Table 2, that the combination of strong world GDP growth (and the resulting strong demand for oil) and strong cohesion of OPEC work as drivers for higher oil prices. The possibility of an immediate connection between conclusions and assumptions is a particular strength of CIB. With most forms of the Cross-Impact analysis this is not possible.
Although the definition of consistent scenarios has to be carried out by a computer for reasons of quantity, no blind trust in a more or less mysterious mathematical procedure is needed by people involved in the analysis. One reason is that the computer only repeats a thousand or a million times a testing step which is understood by those involved. The other reason is that it is possible for everyone involved to check with paper and pencil whether a scenario suggested by the computer which is possibly surprising is in fact consistent, or why a scenario which is rejected by the computer but found plausible by those involved is not consistent as defined by the CIB procedure.

The described analysis allows for some invariance operations with the Cross-Impact matrix which do not affect the selection of the consistent scenarios:

- **IO-1**: the addition of any number in all judgment cells of a judgment group does not change the consistent scenarios. This invariance operation can be used for a standardization of the judgment cells. A convention that fits well with the formulation of the Cross-Impact question, for example, is that the sum of all the entries in a judgment group has to be 0. In Table 2 this convention is already used.
- **IO-2**: the multiplication of all judgment cells in the judgment groups of a descriptor column with a positive number does not change the consistent scenarios.
- **IO-3**: from IO-2 follows that the multiplication of the judgment cells of the whole Cross-Impact matrix with a positive number does not change the consistent scenarios as well.

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Although the set of consistent scenarios in Table 3 reflects the openness of the future and therefore makes the development of contingency strategies possible, the range of possibilities, on the other hand, is noticeably restricted. So there are states that do not appear in any scenario of the example (e.g. world GDP growth >3%), but also states that are uniform in all of the scenarios (oil price 35–50$).

The consistent scenarios particularly stand out from the mass of combinatorially possible scenarios due to their total self-consistency. Yet, it might be useful to take a look at other scenarios too: (i) it can be taken into account that it is in the nature of Cross-Impact judgments that they are uncertain and that in most cases slightly different entries would be justifiable as well. (ii) We have to bear in mind that the chosen set of descriptors excludes all variables of minor relevance. The influence of the excluded variables is weak, but not zero and they might persuade the system to realize states different from the states of maximum impact scores.

That is why such scenarios for which the Cross-Impact balance shows only a slight inconsistency are not completely irrelevant although the analysts’ main focus should be on the consistent scenarios.

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Table 3

Consistent scenarios of the system example “oil price” (the headings of the scenarios are subjective interpretations)

<table>
<thead>
<tr>
<th>Scenario A</th>
<th>Scenario B¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variant B1</td>
<td>Variant B2</td>
</tr>
<tr>
<td>“Conflict and economic disappointment”</td>
<td>“Calm steps ahead”</td>
</tr>
<tr>
<td>World GDP growth</td>
<td>&lt;2%/year</td>
</tr>
<tr>
<td>Borrowing industrial countries</td>
<td>high</td>
</tr>
<tr>
<td>World tensions</td>
<td>strong</td>
</tr>
<tr>
<td>Cohesion of OPEC</td>
<td>strong</td>
</tr>
<tr>
<td>Oil price</td>
<td>35–50$</td>
</tr>
</tbody>
</table>

¹ Scenarios B1 and B2 were interpreted due to their similarity as sub-variants.

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- **IO-2**: the multiplication of all judgment cells in the judgment groups of a descriptor column with a positive number does not change the consistent scenarios.
- **IO-3**: from IO-2 follows that the multiplication of the judgment cells of the whole Cross-Impact matrix with a positive number does not change the consistent scenarios as well.
measure for the inconsistency can be defined as the sum of those points with which the states of a scenario in the impact balances stay behind the states of maximum impact scores. Thus consistent scenarios have always the inconsistency value of zero. The scenario marked in Table 2 has the consistency 4. Fig. 1 shows the frequency distribution of the inconsistency for the scenarios of the example “oil price”. The higher the inconsistency of a scenario is the worse it complies to the picture of the system the experts have formulated in the form of the Cross-Impact matrix. Fixing the limit up to which inconsistency scenarios should still be regarded as relevant needs a case-specific justification—the higher the experts’ uncertainty with the fixing of the Cross-Impact judgments is and the higher the influence of variables is rated that had to be excluded from the analysis due to reasons of effort, the higher the inconsistency limit ought to be chosen.

5. The succession analysis

In Section 4 the consistent scenarios of the CIB analysis were determined by means of enumeration. Yet there is another way which requires a slightly bigger computational effort but in return allows for additional insight. In order to understand the motivation for this procedure we once again take a look at the result of the consistency test for the exemplary scenario in Table 2. We found that there is consistency for all descriptors except for the oil price. For the oil price the impact balances refer to a higher price that fits the assumed scenario. It seems natural to ‘correct’ this consistency mistake by assuming in a further step the same scenario including this higher oil price. But it turns out that this correction does not lead to a consistent scenario. The intervention, as it was to be expected for a complex system, results in changes in other areas causing new inconsistencies.

Table 4 shows a sequence of scenarios in which, beginning with a chosen starting scenario, each of the following scenarios develops from “adjusting” all inconsistent descriptors. The resulting scenario is

4 The case that the highest value is achieved by more than one descriptor state has to be regulated by a convention. Subsequently the convention applies that for equal impact scores the first state on the left (respectively on the top) is chosen. In order to give sense to this convention the otherwise insignificant order of the states ought to be chosen in such a way that the states that are regarded as more plausible are listed first while building the structure of the Cross-Impact matrix.
Table 4
A state succession

<table>
<thead>
<tr>
<th>GDP growth</th>
<th>Borrowing</th>
<th>World tensions</th>
<th>Cohesion OPEC</th>
<th>Oil price</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;2%/year</td>
<td>2–3%/year</td>
<td>&gt;3%/year</td>
<td>Strong</td>
<td>Moderate</td>
</tr>
<tr>
<td>High</td>
<td>Medium</td>
<td>Low</td>
<td>Strong</td>
<td>Moderate</td>
</tr>
</tbody>
</table>

Start scenario
Balances
-3 0
First successor
Balances
-2 0
Second successor
Balances
-2 0

...
called the successor of its predecessor. The result of continuing a succession depends on the Cross-Impact matrix and the chosen starting scenario. Often the succession results in a consistent scenario after a few steps and is stable from this step onwards. That is why the consistent scenarios can be described as attractors of the succession.

From a mathematical perspective the described method is similar to a one-dimensional cellular automaton with discrete values (for cellular automata compare for instance [39] and [40]). But while research mainly deals with cellular automata with many cells and uniform transformation rules, the Cross-Impact succession defines a cellular automaton with relatively few cells with non-uniform transformation rules, however.

The succession should not be interpreted as direct representation of the real dynamics of the system. But as the temporal changes of a system proceed at least in tendency in a way that the quantities go towards the strongest influence, the succession at least offers an idea of the direction in which the system might move from its momentary state.

The usefulness of this formal exercise becomes obvious when one carries out the succession for every possible start scenario and then calculates the frequency of the final states of the process. As a result one gets a combinatorial weight of the consistent scenarios by which this so far homogenous group can now be distinguished with the help of a quality measure. Table 5 shows the combinatory weights for the consistent scenarios of the exemplary system.

The combinatorial weights are no probabilities but they can be interpreted in a similar way. They measure the attractor basins within the space of the scenario which belong to the consistent scenarios as attractors of the Cross-Impact matrix. If there are no other hints showing which of the possible scenarios a system is going to prefer then these weights justify a rational preference for assuming rather a scenario of higher weight than one of lower weight. In the shown example the weights indicate that the three consistent scenarios ought not to be treated equally but that scenario A is of a predominant importance.

Together with the consistency measurement there are two measurements available in the CIB analysis for the assessment of a scenario. They have different meanings which are both important for the assessment of a scenario and are not to be confused: the inconsistency measurement explains how well a scenario corresponds to the expert judgments of the Cross-Impact matrix. The combinatorial weight, on the other hand, tells us to what extent the system might be inclined to adopt this state. With this two-dimensional quality scale CIB offers new possibilities for describing scenarios.

The succession analysis provides even more information. Not in every matrix all scenarios end in a consistent scenario. For the matrix “oil price” in Table 2, for example, this is not the case as one can see by the addition of the weights in Table 5. For 188 out of the 324 scenarios the succession ends in a

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>186</td>
</tr>
<tr>
<td>B1</td>
<td>1</td>
</tr>
<tr>
<td>B2</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5
Combinatory weights of the consistent scenarios (system example “oil price”)

---

5 Transformation rules are uniform when the same rules apply to all cells. This is not the case here as the different values in the Cross-Impact matrix define specific rules for every descriptor.
consistent scenario if it is used as starting scenario. That means that 136 scenarios do not do that and the starting scenario of Table 4 belongs to this group. So there has to be another group of attractors apart from the consistent scenarios. In fact there is only one other kind of imaginable attractor type for the described succession process: cyclic attractors in which the succession after a certain amount of steps goes back to a scenario that has already appeared and then goes through the same sequence again and again. The number of steps between two repetitions is called the period of the cycle. It is at least 2. The theoretical maximum is given by the number of the combinatorial scenarios of the Cross-Impact matrix, but in practice cycles with periods >10 are rarely found. The longer the period of a cycle is, the more the character of the attractor resembles an aperiodic, chaotic dynamic. One cyclic attractor with period 4 and weight 136, shown in Fig. 2, exists for the system example “oil price”. As has been mentioned above the example shown is not the product of a professional discussion about the oil price system; it only functions as a demonstrative example here. It is also to be stressed again that the succession analysis and the resulting cyclic attractors should not be interpreted as an actual dynamic analysis. For this it would be necessary to take into account descriptor-specific time constants, just to mention one point. But the existence of cyclic attractors can be understood as a hint that certain parts of the system are logically connected in a way that can give rise to instationarity.

However, there are systems in which an interpretation of instationary attractors is not appropriate. This includes systems in which the states of the descriptors have a time-related meaning themselves as in the example in Table 9 that will be described later. In this case the cyclic attractors of a Cross-Impact matrix have to be ignored and only the stationary attractors, that means the consistent scenarios, are to be used.

Not all descriptors inevitably take part in a cycle. In the example of Fig. 2 the descriptors ‘world GDP growth’, ‘borrowing’ and ‘world tensions’ are stationary. The descriptors ‘cohesion of OPEC’ and ‘oil price’, on the other hand, make each other oscillate. Only with this cyclic attractor the resulting picture of the exemplary system “oil price” is complete. It answers the unanswered question from Table 3, whether strong world GDP growth rates can occur. The answer is yes; and strong growth introduces an element of instationarity into the system. So the complete solution table for the considered example appears as
shown in Table 6. The low weight of the scenarios B1 and B2 indicates that these scenarios are possible but presumably harder to achieve and to stabilize.

6. An application example

Meanwhile experiences with the use of the CIB method have been gathered in several projects [41–45], one is outlined in the following. In 2003 the Center for Technology Assessment and the Institute for Social Sciences of the University of Stuttgart took part using this method in a project led by the Forum for Energy Models and Energy-Economic Systems Analysis in Germany (FEES). In the project the role of German power generation for the European climate protection in 2030 was to be examined.

The Cross-Impact analysis was carried out in the following steps [45]: first of all an expert panel from different research institutions was established. This panel structured the problem during a working session and a suitable set of descriptors was developed (Table 7). This task could be carried out quicker than in normal scenario processes because all experts were well familiar with the procedure and because the question was already prestructured. As a next step the Cross-Impact matrix was drawn up in a 1-day workshop. For this all relevant interrelations were discussed. On this occasion an agreement could be reached for nearly all Cross-Impact judgments. The few exceptions in which an expert dissent remained despite the exchange of all arguments were registered as a vote for a sensitivity analysis. In a further 1-day workshop the Cross-Impact matrix was again discussed and revised with the help of temporary analysis results.

The system described in Table 7 is heterogeneous in an important way. Part of the descriptors interacts with the other descriptors in a way that is mathematically barely to model and is therefore only accessible for the analysis in the form of expert judgments. An example for this is the descriptor “innovation impulses and effects”. The whole point of the Cross-Impact analysis is that such descriptors can be analyzed as well. But that does not mean that all descriptors are of that kind. So CO2 emissions and the power generation costs are deducible from other descriptors by way of calculation. In these cases it would introduce an avoidable inaccuracy into the process to use expert judgments.

But CIB offers a chance to deal appropriately with those mixtures of soft and hard quantities. That is why Cross-Impact judgments for the connections that can be mathematically represented were not estimated in this project. Instead the mathematical connection was evaluated in a value table. Afterwards
it was determined with the help of linear programming which Cross-Impact matrix entries represent the 
calculated value table on the condition of a minimal sum of points. Because of that the columns of these 
descriptors in the Cross-Impact matrix could be filled with calculated values (compare Box 1). In this 
way CIB works as an integrative analysis platform in which estimated, qualitative connections as well as 
mathematically comprehensible connections can be inserted. Besides, valuable discussion time is 
concentrated on those parts of the matrix which are actually only accessible with the help of expert 
judgments. The final Cross-Impact matrix as provided by the experts was analyzed in several ways. The 
基本 evaluation is the listing of the consistent scenarios with their combinatorial weights (Table 8). This 
resulted in 6 scenarios which, for instance, can be sorted according to their statements about the effects 
of certificate prices (Fig. 3).

A further approach to the analysis of the matrix are intervention analyses. Here an additionally strong 
exogenous impulse is assumed for the benefit of a descriptor state, e.g. “wind power 130 TWh”. This is 
realized by introducing an additional descriptor “intervention” into the Cross-Impact matrix which only 
one very high positive entry in the judgment cell “intervention has an impact on wind power 130 TWh” 
and is otherwise set to 0. By a comparison of the weighted frequency of all the other descriptor states in

### Table 7

<table>
<thead>
<tr>
<th>Descriptor</th>
<th>States in year 2030</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price of CO₂ emission certificates</td>
<td>0 €/t CO₂, 10 €/t CO₂, 20 €/t CO₂</td>
</tr>
<tr>
<td>Wind power</td>
<td>Onshore (20 TWh per annum), Onshore (41 TWh per annum), Onshore+offshore (130 TWh per annum)</td>
</tr>
<tr>
<td>Power plants (Power generation except for wind and import)</td>
<td>75% Coal, 20% Natural gas, 5% Renewable, 67% Coal, 20% Natural gas, 13% Renewable, 35% Coal, 60% Natural gas, 5% Renewable, 31% Coal, 56% Natural gas, 13% Renewable</td>
</tr>
<tr>
<td>Import</td>
<td>0 TWh per annum, 50 TWh per annum, 100 TWh per annum, 150 TWh per annum</td>
</tr>
<tr>
<td>Cost of power generation</td>
<td>3.8 € c./kWh, 4.5 € c./kWh, 5.1 € c./kWh</td>
</tr>
<tr>
<td>Consumer efficiency</td>
<td>Electricity consumption: baseline, Electricity consumption: baseline—5%, electricity consumption: baseline—10%</td>
</tr>
<tr>
<td>Innovation impulses and effects</td>
<td>Low, Medium, High</td>
</tr>
<tr>
<td>CO₂ emissions of power plants</td>
<td>Baseline, Baseline—100 Mt CO₂ per annum, Baseline—200 Mt CO₂ per annum</td>
</tr>
<tr>
<td>Employment effect (on energy economy)</td>
<td>Negative, Neutral, Positive</td>
</tr>
</tbody>
</table>
Box 1. Integration of mathematically tangible interrelations in CIB

CIB matrices can also be used for representing exact interrelations, they work as a glass clockwork. So the following matrix realizes the addition of two values $x_1$ and $x_2$ to a value $x_3$:

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_3$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The list of consistent scenarios of this matrix is exactly the value table of the addition. Thus also more complex mathematical equations can be represented. The Cross-Impact values suitable for that can be found by first establishing a value table of the equation. For the descriptors $1\ldots n$ (with $z_i$ states each) which are part of an equation for the descriptor $m$ the value table with $p = \Pi z_i$ rows shall be:

<table>
<thead>
<tr>
<th>Descriptor no.</th>
<th>$1$</th>
<th>$2$</th>
<th>$\ldots$</th>
<th>$n$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>1</td>
<td>1</td>
<td>$\ldots$</td>
<td>1</td>
<td>$m_1$</td>
</tr>
<tr>
<td>$2$</td>
<td>1</td>
<td>1</td>
<td>$\ldots$</td>
<td>1</td>
<td>$m_2$</td>
</tr>
<tr>
<td>$z_1$</td>
<td></td>
<td></td>
<td>$\ldots$</td>
<td>$z_n$</td>
<td>$m_p$</td>
</tr>
</tbody>
</table>

$m_k$ is the number of the discrete state of the descriptor $m$, which is for the value combination of the descriptors $1\ldots n$ in row $k$ the best possible (generally in rounded figures) representation of the equation. The state numbers of the descriptors $1\ldots n$ form the vector $w_{ki}$ in row $k$ of the value table. The Cross-Impact matrix elements in the column of the descriptor $m$ have to fulfill the following conditions for every $k$ so that the consistent CIB scenarios and the value table harmonize:

$$\sum_i C_{im}(w_{ki}, m_k) \geq \sum_i C_{im}(w_{ki}, l) + 1 \quad \text{if} \; l \neq m_k$$  \hspace{1cm} (a)

Each solution of (a) in principle fulfills the purpose (the conversion of the value table) equally well. However, solutions with the smallest possible number of points are the clearest and therefore the easiest ones to interpret. Therefore the set of Cross-Impact matrix elements must be preferred that fulfills (a) and for which is also true:

$$\sum_{l,k,l} |C_{im}(k, l)| = \min$$  \hspace{1cm} (b)
the consistent scenarios that arise with and without this impulse, one can judge which quantities would react sensitively and which ones would react rigidly to a particular intervention into the matrix from the outside. What is useful about this is that the logic of the Cross-Impact matrix does not only take into account the direct but also all indirect effects over two or more additional quantities for this.

A further possibility for an analysis which is based on this is represented by inverse intervention analyses. In an inversion of the question just described, it can be analyzed at which point of the system one would have to intervene in order to further a particular descriptor state in the best possible way. For this, intervention analyses for all other descriptor states are carried out and in each case it is made a note of how strongly the frequency of the target state changes. Fig. 4 shows the result of this analysis for the target state “neutral effect of employment”.

The results must be interpreted in a way that only few interventions are suitable to ensure a neutral effect on employment. Many interventions with a helpful direct effect become counterproductive because of indirect effects. Only high certificate prices, high wind power usage and high power imports (which have been assumed to be mainly renewable power imports with a corresponding vortex effect on the production of domestic renewable energy) promise a success that is worth mentioning according to the Cross-Impact judgments assessed.

7. A system-theoretical foundation of CIB analysis

From a pragmatical point of view the definition of the Cross-Impact judgments and a heuristic evaluation procedure designed for them, as described in Section 4, are sufficient. The experts questioned have an adequate idea about the type of judgments which are expected from them, of the logic on account of which conclusions are constructed from these judgments and of the interpretations which these results allow. Nevertheless, it is good when a newly recommended Cross-Impact method offers more than that because on the one hand interpretation questions can arise during the practical work that cannot be clearly decided on a pragmatic level and that need the backing of a theoretical foundation; and on the other hand the family of Cross-Impact methods is sometimes confronted with the criticism to produce partly arbitrary results with its use of heuristic procedures. But this criticism becomes unjustified if the method can be backed up theoretically. Probabilistic Cross-Impact methods look for

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6 Possible employment losses due to these measures in other sectors of the economy because of higher electricity prices were made subject of discussions among the experts but are not the object of the Cross-Impact analysis in the chosen system definition.
this foundation in the theory of probabilities. Non-probabilistic Cross-Impact approaches like the one at issue should be justifiable with reference to the mathematical systems theory. One of the most general possibilities to describe time-varying systems are systems of coupled first-order differential equations

$$\dot{x} = N(x, t)$$  \hspace{1cm} (1)

in which \(x\) is the vector of the dynamic state variable with the components \((x_1, x_2, \ldots, x_n)\). \(N\) represents a vector of functions that can also be non-linear and can be described as system forces because they, similar to mechanical forces, determine the speed and direction of motion of the system (Fig. 5). Equations of the type Eq. (1) describe an enormous variety of phenomena and form the backbone of mathematical analysis of dynamic systems in physics (e.g. in mechanics), chemistry (e.g. in the form of reaction equations), engineering (e.g. in electrodynamics) and biology (e.g. within the scope of population dynamics). System dynamics models correspond to this concept also. Spatiotemporal processes can be transferred into the form of Eq. (1) by expanding the space-dependent variables.

<table>
<thead>
<tr>
<th>Price of certificates</th>
<th>Wind power</th>
<th>Power plants</th>
<th>Import</th>
<th>Generation costs</th>
<th>Efficiency</th>
<th>CO₂ emissions</th>
<th>Innovations</th>
<th>Employment effect</th>
<th>Scenario weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 €</td>
<td>20 TWh</td>
<td>Coal</td>
<td>0 TWh</td>
<td>3.8 ct</td>
<td>0%</td>
<td>−0 Mt</td>
<td>Low</td>
<td>Neutral</td>
<td>11.664</td>
</tr>
<tr>
<td>10 €</td>
<td>41 TWh</td>
<td>C.+Ren.</td>
<td>50 TWh</td>
<td>4.5 ct</td>
<td>5%</td>
<td>−0 Mt</td>
<td>Medium</td>
<td>Negative</td>
<td>11.601</td>
</tr>
<tr>
<td>20 €</td>
<td>130 TWh</td>
<td>G.+Ren.</td>
<td>50 TWh</td>
<td>5.1 ct</td>
<td>10%</td>
<td>−200 Mt</td>
<td>High</td>
<td>Neutral</td>
<td>10.260</td>
</tr>
<tr>
<td>20 €</td>
<td>41 TWh</td>
<td>Gas</td>
<td>50 TWh</td>
<td>4.5 ct</td>
<td>5%</td>
<td>−100 Mt</td>
<td>Medium</td>
<td>Negative</td>
<td>1,035</td>
</tr>
<tr>
<td>20 €</td>
<td>41 TWh</td>
<td>Gas</td>
<td>50 TWh</td>
<td>4.5 ct</td>
<td>10%</td>
<td>−100 Mt</td>
<td>Medium</td>
<td>Negative</td>
<td>369</td>
</tr>
<tr>
<td>10 €</td>
<td>20 TWh</td>
<td>Coal</td>
<td>0 TWh</td>
<td>3.8 ct</td>
<td>0%</td>
<td>−0 Mt</td>
<td>Low</td>
<td>Neutral</td>
<td>63</td>
</tr>
</tbody>
</table>

Fig. 3. Effects of the price of CO₂-certificates on the descriptors. The bar lengths represent the combinatoric weight of the scenarios.
according to suitable space functions. Eq. (1) also forms the basis for the consideration of random influences if the equations are extended to Langevin-Equations by adding fluctuating terms [46].

Autonomous systems, i.e. systems which do not show external time dependency of the system forces in Eq. (1), can have states of equilibrium in which all variables have adjusted themselves in such a way that their interactions are balanced and that the system can permanently stay in this state. These states of equilibrium are, provided they exist, characterized by the condition

$$\dot{x} = 0$$

and are referred to as stationary states of the system. Together with Eq. (1) this results in

$$N(x) = 0$$

as an equation for the equilibrium states of autonomous systems.

Fig. 4. Inverse intervention analysis on the state “neutral employment effect”. CP: CO₂ certificate price, WP: wind power, PP: power plants, Imp: import, GC: generation costs, Eff: consumer efficiency, Inn: innovation impulses and effects.

Fig. 5. Force field and trajectory of an autonomous system with two variables $x_1$ and $x_2$. The force field indicates the strength and the direction of the system forces at every point of the system space. The curve of successive states (trajectory) shows the actual dynamic evolution of the system. The trajectory always runs parallel to the local direction of the forces.
The solution of this equation for \( x \) represents the state of equilibrium and is called stationary solution. We will not deal with the determination of the stability characteristics of states of equilibrium. Readers interested in this problem should consult relevant textbooks.

One cannot generally assume the lack of outward time-dependent influences in technological-political-social systems. But the concept of equilibrium is also relevant for non-autonomous systems. If the external influences change slowly enough the system can adjust to a stable equilibrium again after each change, provided that a stable equilibrium still exists. The development of the system can then approximately be understood as a succession of stable states of equilibrium, which is called a quasi-stationary evolution.

A special case which is of importance for our purpose is represented by pair-force systems. The system force onto a quantity in this system consists of the superposition of interactions in pairs with this quantity. The system equation then reads:

\[
\dot{x}_i = N_i = \sum_j f_{ij}(x_i, x_j). \tag{4}
\]

In this case the \( f_{ij} \) are arbitrary functions of \( x_i \) and \( x_j \) which will be assumed as continuous functions from now on. By rearrangement of the terms in Eq. (4) it can always be achieved that the sum does not contain any diagonal elements \( f_{ii} \). This will also be assumed from now on. For stationary or quasi-stationary states of equilibrium the following applies:

\[
\sum_j f_{ij}(x_i, x_j) = 0. \tag{5}
\]

This kind of system description is strongly determined by a mathematical point of view and the question arises whether there is really a connection to the very pragmatic and qualitatively oriented Cross-Impact analyses. It indeed exists. In order to find it we introduce the antiderivatives \( F_{ij}(x_i, x_j) \) of the pair-forces

\[
f_{ij} = \frac{\partial F_{ij}}{\partial x_j}. \tag{6}
\]

Together with Eq. (5) the following applies:

\[
\sum_j \frac{\partial F_{ij}}{\partial x_i} = \frac{\partial}{\partial x_i} \sum_j F_{ij}(x_i, x_j) = \frac{\partial}{\partial x_i} \phi_i(x) = 0. \tag{7}
\]

While the pair-forces \( f_{ij} \) describe which forces on the system quantity \( i \) result from the pair interaction between the quantities \( i \) and \( j \), the \( F_{ij} \) have to be interpreted as accumulated system forces and are in a certain way close to the physical concept of energy. The equilibrium condition has taken such a form that the derivatives of certain functions \( \phi_i \) have to disappear. That is equal to the fact that these functions themselves take on an extreme value at the points of equilibrium. So the rule for finding equilibrium positions of a dynamic system of the type Eq. (4) is:

- Search for a set of values for the variables \( \{x_i\} \) so that all sums \( \phi_i = \sum_j F_{ij}(x_i, x_j) \) are simultaneous extreme values concerning a change of \( x_i \).
Now the link to the CIB analysis emerges. In CIB the rule for finding consistent scenarios is:

- Search for a set of descriptor states \{z_i\} so that all impact scores \(s_i = \Sigma_j C_{ij}(z_j, z_i)\) are each maximized with regard to the discrete value space of \(z_i\).

Thus if for a Cross-Impact matrix the matrix elements are chosen according to

\[ C_{ji}(k, l) = F_{ij}(x_{il}, x_{jk}) \]  \hspace{1cm} (8)

and \(x_{jk}\) is the value of the state \(k\) of descriptor \(j\), then the impact scores of descriptor \(i\) correspond to function \(\phi_i\) in Eq. (7) and the consistent scenarios of the CIB analysis are approximately equilibrium states of the system shown in Eq. (4). ‘Approximately’ because the taking over of the accumulated forces into the Cross-Impact matrix by Eq. (8) is only equivalent to a “scanning” of the functions \(F_{ij}\) in steps (compare Fig. 6). The smaller the interval steps of a descriptor are chosen, the more reliable is this scanning. In the limit of infinite fine intervals in the Cross-Impact matrix the identity between consistent scenarios and equilibrium states is absolute.

In slowly changing non-autonomous systems the \(f_{ij}\) and therefore also the \(F_{ij}\) and the \(C_{ij}(k, l)\) are slowly time-dependant. The expert judgments and the resulting consistent scenarios in this case have to refer to a definite time. A dynamic element can be introduced into the CIB analysis by additionally estimating which matrix elements must be expected to change with respect to time and in what way this will happen. The CIB analysis can then be carried out in time layers and provides a time series of scenarios.

In some cases the CIB analysis selects only a part of the existing equilibrium states for the determination of the consistent scenarios. The reason for this is that all extreme values of the functions \(\phi_i\) are basically suitable as elements of an equilibrium, that means maxima as well as minima and horizontal tangents. CIB regards only absolute maxima as useful parts of a consistent scenario. Scenarios which contain for instance minimal impact scores are dismissed as not useful due to the meaning behind the Cross-Impact judgments; this is to be seen as an additional interpretative act of the CIB analysis. On the other hand systems theory differentiates between stable and unstable equilibrium states, considering only the stable one to be relevant in general. The stability is determined with the help of a perturbation calculation (compare, e.g. [46]). It will have to be the object of further research to establish correspondences between the stability criterion of systems theory and the criterion of maximal consistency in CIB.

So, from a mathematical point of view, CIB is an approximation to search for equilibrium states of a class of systems which is defined by Eqs. (4) and (5), respectively. In other words, Eqs. (4) and (5) represent the variety of systems which are accessible to a CIB treatment according to the described system-theoretical interpretation. This variety is quite far-reaching as the \(f_{ij}\) may be any continuous functions of their arguments. Complete universality is not gained by Eqs. (4) and (5), however. The limitation on systems whose behaviour is determined by pair interactions which becomes explicit in Eqs. (4) and (5) is not a specific quality of CIB, though, but is constitutional for the Cross-Impact concept and in one or another way is of limiting effect in all methodological forms of this concept.

The described correspondence of mathematical system descriptions and CIB does not only provide a theoretical background for CIB but is also of practical importance: Eqs. (6) and (8) provide another and very systematic method for the translation of mathematizable knowledge about system parts into the
Cross-Impact language besides linear programming (compare Box 1). A simple demonstration is provided by the addition matrix in Box 1. The addition of $x_1$ and $x_2$ can be expressed by

$$x_1 + x_2 - x_3 = 0.$$  

(9)

This equation corresponds to the structure of Eq. (5) and therefore can be evaluated by a CIB matrix. Eq. (9) can be expressed by the pair-force functions

$$f_{31} = x_1 - x_3 / 2 \quad f_{32} = x_2 - x_3 / 2 \quad \text{all other } f_{ij} = 0$$  

(10)
Eq. (11) shows antiderivatives of these pair-force functions (cf. Eq. (6)):

\[ F_{31} = x_1x_3 - \frac{x_3^2}{4} \quad F_{32} = x_2x_3 - \frac{x_3^2}{4} \quad \text{all other } F_{ij} = 0. \]  

(11)

Insertion of the interval values 0, 1, and 2 for \( x_1, x_2, \) and \( x_3 \) into Eq. (11) yields directly the Cross-Impact values of the addition matrix.\(^7\)

Of course, one would not begin a Cross-Impact analysis in practice searching at first for functional relations in the form suggested by Eqs. (4) and (5) for all variables and then converting them with the help of Eq. (8) into a Cross-Impact matrix. Problems which allow this are better resolved directly in a purely mathematical way. For typical problems which are suitable for a Cross-Impact analysis the relations for at least a part of the variables are not known with the precision of Eqs. (4) and (5), but can only be estimated. However, the described theoretical foundation is important for these cases as well, as it defines specifically at which quantities the Cross-Impact question which has been pragmatically formulated in Section 4 really aims.

8. Classifying the CIB analysis

Although the CIB method introduced here represents a new Cross-Impact procedure, many of its roots can be found in the family of Cross-Impact methods. The first relation to be mentioned is that to the Cross-Impact concept itself, which was demonstrated by Gordon and Helmer in the *Future* game, a promotional gift from Kaiser Aluminium, and then described by Gordon and Hayward [12].

The approach used by CIB to characterize system states by means of descriptors which are classified by discrete states and value intervals goes back to BASICS [22], whereas the CIB algorithm differs completely from BASICS.

The structural element to combine Cross-Impacts by sums can be found also at simulation languages as KSIM [25] and QSIM2 [26]. This relationship opens in principle the chance, to transform KSIM and QSIM2 models in a rudimentary way into CIB models and vice versa. Some of the Cross-Impact methods deal—as the original approach by Gordon, Helmer and Hayward—with the event sequences which can arise from the mutual influences of the events. Others rather concentrate on the analysis of a certain point of time in the future, in which the causal conditionings of the preceding event sequences are reflected implicitly (e.g. SMIC74 [15]). CIB in the form that has been described so far rather belongs to the second group. The overwhelming part of Cross-Impact approaches is probabilistic. The reasons given for this are among others that the future is not unambiguous and scenario analyses should alert the decision maker to a spectre of possible developments to support him or her in developing contingency strategies. CIB is not probabilistic in the form described so far, but nevertheless reaches this target: the procedure possesses an implicit tendency to show multiple futures. It differs from probabilistic procedures only in the fact that the branches of the event tree, which stand between present and future, are not interpreted as interventions by chance but by decisions. Openness of the future arises in both cases.

In the preceding paragraphs the most important features for distinguishing different Cross-Impact approaches have been sketched and CIB has been classified accordingly. Doing this, the reservation \(^7\) The matrix values in the box were multiplied by 4 in order to get integer numbers. This invariance operation IO-2 (cf. Section 4) does not affect the CIB analysis.
described so far’ was used quite often. The reason for this is that CIB possesses very flexible structures, which enable the analyst to apply different methodical paradigms based on CIB. In that way, CIB can form a bridge between the different methodological branches of the Cross-Impact family. So CIB can also be run as a probabilistic model by varying the succession described in Section 5. Instead of choosing in a deterministic way the state of the highest impact score for each step, the choice can be done randomly according to state probabilities,

$$p_k \propto z^{S_k}$$

(12)

$S_k$ representing the impact score of the state $k$. The succession then corresponds to a stochastic cellular automaton. The parameter $z > 1$ shows how much the probability rises if one state shows an impact score 1 point higher than another state. In the limit of high $z$ there results a deterministic succession, $z = 1$ describes the limit where random influences are so strong that the system-endogenous influences do no longer have any effect: the system is completely “noisy”. Different approaches than Eq. (12) appear to be possible as well. The reasons for a stochastic succession are (i) the less important quantities of influence that are not taken into account for the choice of the system descriptors can at least cause disturbances and so distract the system from realizing the scenarios of the highest consistency; (ii) expert judgments are imprecise. It can therefore happen that the points are not allocated in a way that would correspond to the reality in the best possible way. Then $z$ measures the probability whether judgments were given that are deviating wrongly by one point: is this probability high, a small value must be chosen for $z$.

Due to the stochastic succession, probabilities are assigned to the scenarios instead of combinatorial weights. The probabilities can be determined by means of a simulation.

Bridges can be built to the group of Cross-Impact methods as well which aim at the investigation of event sequences. To achieve this, the descriptors have to be defined as events and their states describe time intervals for the occurrence time instead of value intervals. Table 9 shows a simple example of a judgment section for this approach.

The examples demonstrate that CIB does not only offer varied analytical possibilities in its basic form but also possesses a considerable methodological flexibility in order to be able to adapt to different methodological paradigms. Therefore, CIB is less a representative of a certain branch of the Cross-Impact family than a possibility to integrate the advantages of different branches into a unified analytical instrument, which can be developed specifically from case to case.

9. Closing remarks

Godet identified the reduction of prognostic problems to quantifiable problem parts as an important reason for the frequent prognosis failures in the past [7]. Analyses of the future therefore often
manoeuvre between Scylla and Charybdis: a comprehensive definition of the problem can mean a renunciation of computational models and consequently a serious loss of stringency. On the other hand, to keep the analysis narrowly to mathematically treatable concepts can blend out crucial elements of the systemic interplay. Which danger in each case is the most serious one can only be judged for every single instance separately. If one comes to the conclusion that a view of the problem that goes farther than can be reproduced mathematically is inevitable, Cross-Impact analyses with their systematic capturing and processing of structure-free information can be the method of choice. An important additional benefit of Cross-Impact analyses is often that the participating experts are moved to give explicit reasons for their judgments, that differences between the experts’ judgments become obvious and can be discussed and that even for experienced experts a new point of view can be created and their understanding of the system can be consolidated.

The special advantages of the method, however, have to be paid for by specific disadvantages as well: as (i) Cross-Impact methods are based on expert judgments, which have to be made separately for each pair interaction of the system, the number of descriptors that can be taken into account is limited due to practical reasons. Therefore only those systems are accessible for a Cross-Impact analysis for which the target of a qualitative understanding can be reached with a moderate number of system variables. For the same reason Cross-Impact analyses can only create rough scenarios. (ii) The exclusive reliance on expert judgments not only with respect to the data (as it is also the case with computational models), but also with respect to the logical structure of a system means that the results of the analysis must be interpreted keeping in mind the uncertainty of such assessments. An arbitrariness of the Cross-Impact analysis results can only be avoided if the collected expert judgments are more than the result of little reflected guessing.

The new Cross-Impact method (CIB) proposed in this paper possesses some advantages which can be of use when applying the method: (i) the simplicity of its fundamental logic means high transparency even for participants without deep mathematical training and so promotes the acceptance of the method and the results. (ii) It only demands judgments from the experts about the relations between the system elements, i.e. about the system structure, and avoids the assessment of quantities which require a ‘mental integration’ of the system by the experts, which many other methods necessitate. (iii) It makes possible the systematic integration of quantifiable parts of correlations, as far as they are known, and by that provides an integrative analytical basis for mathematizable and non-mathematizable problem parts. (iv) It possesses highly flexible structures, which make it possible to convert different methodological paradigms within the CIB-methodology. (v) The method can be substantiated using a system-theoretical background and thus avoids the element of arbitrariness about which heuristic Cross-Impact methods have been criticized earlier.

Despite these favorable qualities it is necessary to differentiate this method from problems for which other methods have been more specifically created. Problems that allow a theory-based or empirically founded mathematical formulation should of course be analyzed with the help of computational models. Nevertheless, CIB analyses can make a valuable contribution here by offering a preparatory environment analysis or by promoting the analysts’ understanding of the system through an accompanying reflective process. With problems allowing only speculative and vague statements even on the level of expert judgments one should possibly decide against an analysis in general for reasons of intellectual honesty. For analytical tasks referring especially to questions about the sequence of event chains CIB does offer an approach as has been shown. But CIB is not specialized in these questions, which is the reason why possibly other Cross-Impact methods should be applied rather than CIB. Despite these limitations CIB appears to be a promising new method for a wide area of multidisciplinary scenario applications.
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